PROJECTED WRITTEN NOTES FROM THE M408D LECTURE ON Tuesday, January 16, 2024, ON THE M408D CANVAS COURSE, the Class Syllabus, and

Integration using u-substitution and on the the method of Integration called "Integration by Parts".

CLASS #1

M408D SPRING 2024   SEQUENCES, SERIES AND MULTI-VARIABLE CALCULUS

UNIQUE #: 53235 / 53240   T, TH 11:00 AM – 12:30 PM   GSB 2.126

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"M 408D" must be included in the Subject.

Teaching Assistant :

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Office Hours:

M: 2:30 pm – 4:00 pm

TH: 1:30 pm – 3:00 pm

in PMA 12.116

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TEXTBOOK: CALCULUS : Early Transcendentals (9th Edition) by James Stewart.

Each student automatically has access to a digital eBook through the Longhorn Textbook Access Program (LTA) and will be charged a nominal fee unless he/she opts out of the LTA before the 12th class day.

WHERE TO GO FOR THE DISCUSSION SESSIONS

If your unique # for this class is 53235;
The Discussion sessions are in PMA 6.104, 11am-12noon

If it is 53240,
The Discussion sessions are in JGB 2.218,
4pm-5pm

A Definite Integral, like $\int_0^2 x^2 dx$, is a number. Here, $\int_0^2 x^2 dx = \left[\frac{1}{3}x^3\right]_0^2 = \frac{1}{3}8 - \frac{1}{3}0$

$$\int_0^2 x^2 dx = \frac{8}{3}$$

The Integrand!

An indefinite integral, like $\int x^2 dx$, is the FAMILY OF ALL FUNCTIONS That have the integrand as their derivatives!

$$\int x^2 dx = \frac{1}{3}x^3 + C$$

Integration Using u-substitution:

Consider the indefinite integral:

$$\int (x^4+x)^8 \underbrace{(4x^3+1)\,dx} = \int u^8\,du = \frac{1}{9}u^9 + C$$

$$= \frac{1}{9}(x^4+x)^9 + C$$

Let $u = x^4 + x$

$du = \underbrace{(4x^3+1)\,dx}$

Ex: $\int \cos^9 x \,\underbrace{\sin x\,dx} = -\int u^9\,du$

$$= -\frac{1}{10}u^{10} + C$$

Let $u = \cos x$

$du = -\sin x\,dx$

$\underline{\sin x\,dx} = -du$

$$= -\frac{1}{10}\cos^{10} x + C$$

Problem: FIND $\int x^8 \ln x \, dx$

$$\left[\begin{array}{l} \cancel{u = x^8 \quad dv = \ln x \, dx} \\ u = \ln x \quad dv = x^8 dx \end{array}\right.$$

$u = \ln x \leftrightarrow dv = x^8 dx$

$du = \frac{1}{x} \leftrightarrow v = \frac{1}{9}x^9$

$$\int x^8 \ln x \, dx = \frac{1}{9}x^9 \ln x - \int \left(\frac{1}{x}\right)\left(\frac{1}{9}x^9\right) dx$$

$$= \frac{1}{9}x^9 \ln x - \frac{1}{9}\int x^8 dx$$

$$= \frac{1}{9}x^9 \ln x - \frac{1}{9}\left(\frac{1}{9}x^9\right) + c$$

$$= \frac{1}{9}x^9 \ln x - \frac{1}{81}x^9 + C$$

$$= \frac{1}{9} \cdot x^9 \left(\ln x - \frac{1}{9}\right) + C$$

"u-sub with a definite integral"

$$\int_0^1 (x^3+2)^4 \underbrace{x^2 dx} = \frac{1}{3}\int_2^3 u^4 \, du$$

let $u = x^3+2$

$du = 3\underbrace{x^2 dx}$

$x^2 dx = \frac{1}{3} du$

when $x=0$, $u=2$

when $x=1$, $u=3$

$$= \frac{1}{3}\left(\frac{1}{5}u^5\right)\Big]_2^3$$

$$= \frac{1}{15}3^5 - \frac{1}{15}2^5$$

$$= \frac{243}{15} - \frac{32}{15} = \frac{211}{15}$$

$$= 14\frac{1}{15}$$

$$\int (x^3+2)^4 x^2 dx = \frac{1}{3}\int u^4 du$$

$$= \frac{1}{15}u^5 + C$$

$$= \frac{1}{15}(x^3+2)^5 + C$$

$$\int_0^1 \sim dx = \frac{1}{15}(x^3+2)^5\Big|_0^1$$

Integration by PARTS

The "PARTS" Rule: $\boxed{\int u\,dv = uv - \int v\,du}$

when $u = f(x) \longleftrightarrow dv = g'(x)dx$

$du = f'(x)dx \longleftrightarrow v = g(x) = \int dv = \int g'(x)dx$

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

$$\int xe^x dx = \int u\,dv = uv - \int v\,du$$

$u = x \longleftrightarrow dv = e^x dx$

$du = 1dx \longleftrightarrow V = e^x$

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C$$

$$\int xe^x dx = e^x(x-1) + C$$

(Applying "PARTS" TWICE)

$$\left[\begin{array}{l}\int u\,dv = uv - \int v\,du \\ \int w\,dz = wz - \int z\,dw\end{array}\right.$$

Problem: $\int x^2 \cos x\,dx$

$$u = x^2 \qquad dv = \cos x\,dx$$
$$du = 2x\,dx \qquad v = \sin x$$

$$\int x^2 \cos x\,dx = x^2 \sin x - \int 2x \sin x\,dx$$
$$= x^2 \sin x - 2\int x \sin x\,dx$$

$$w = x \qquad dz = \sin x\,dx$$
$$dw = 1 \qquad z = -\cos x$$

$$= x^2 \sin x - 2\left(-x\cos x - \int (-\cos x)\,dx\right)$$
$$= x^2 \sin x + 2x\cos x - 2\int \cos x\,dx$$

$$\int x^2 \cos x\,dx = x^2 \sin x + 2x\cos x - 2\sin x + C$$